

## Block Designs Design Control

Completely Random

I	II	III	IV
Trt 2	Trt 1	Trt 4	Trt 1
Trt 3	Trt 3	Trt 1	Trt 4
Trt 3	Trt 2	Trt 1	Trt 4
Trt 4	Trt 3	Trt 2	Trt 2

$$Y_{ij} = \mu + T_i + \varepsilon_{(ij)}$$

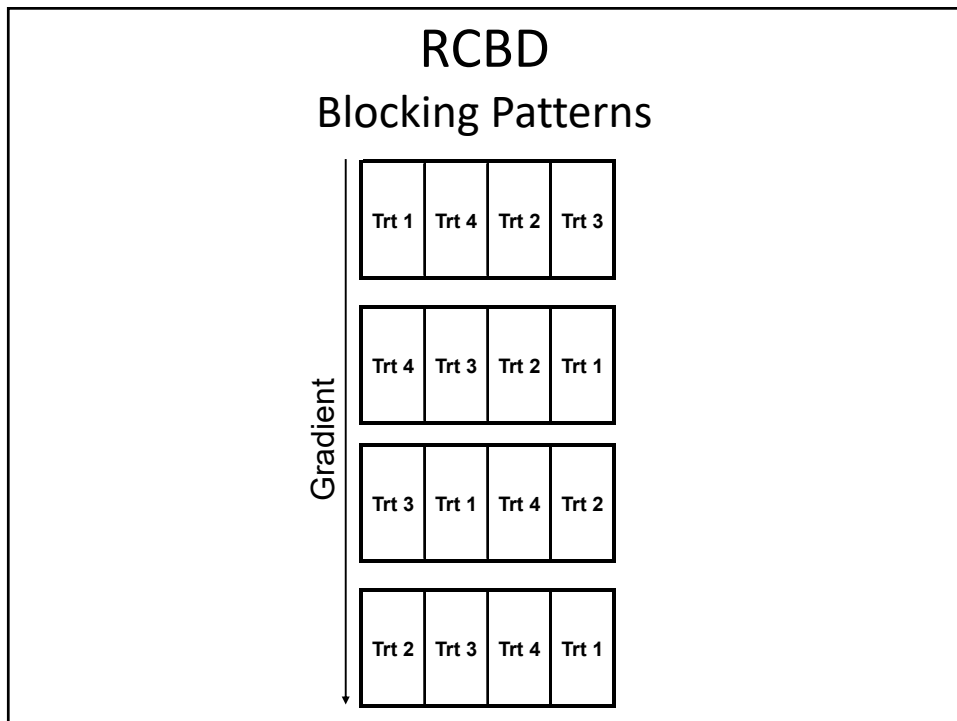
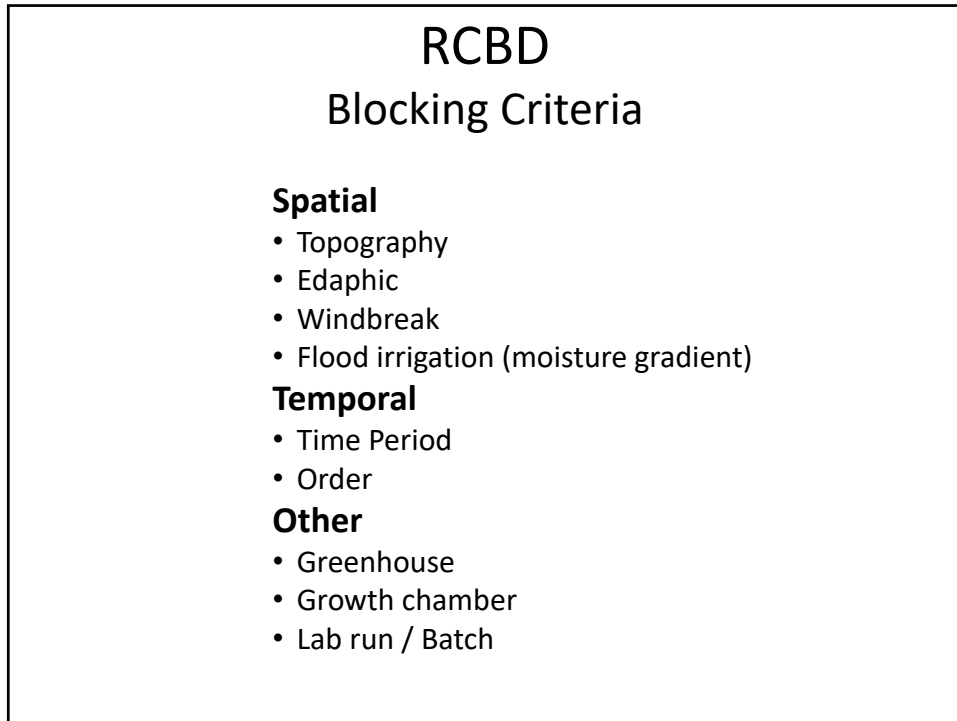
Blocked

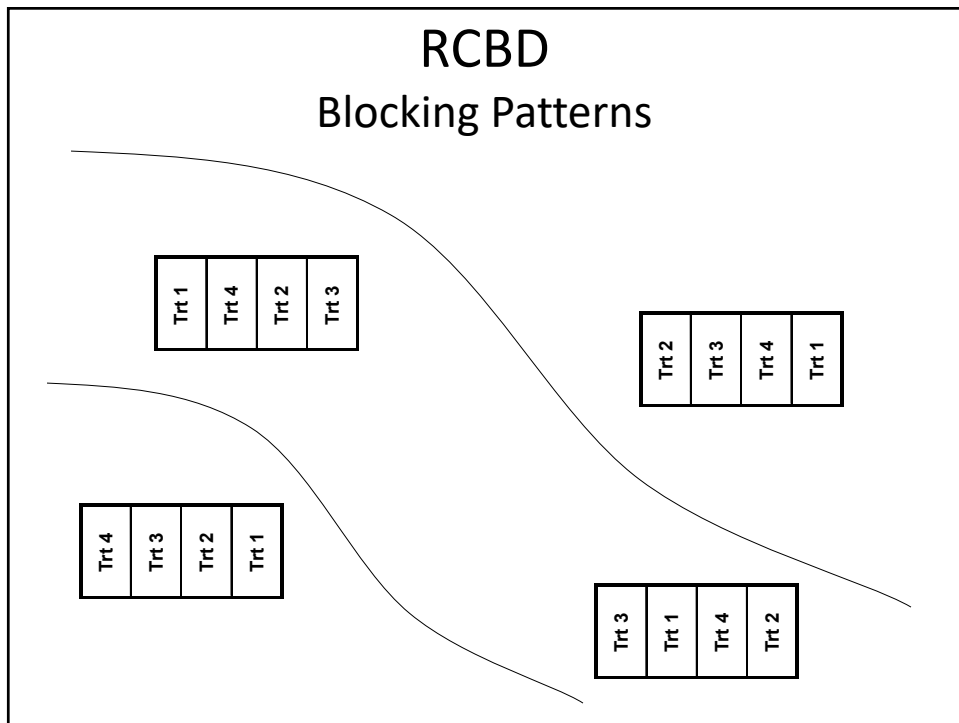
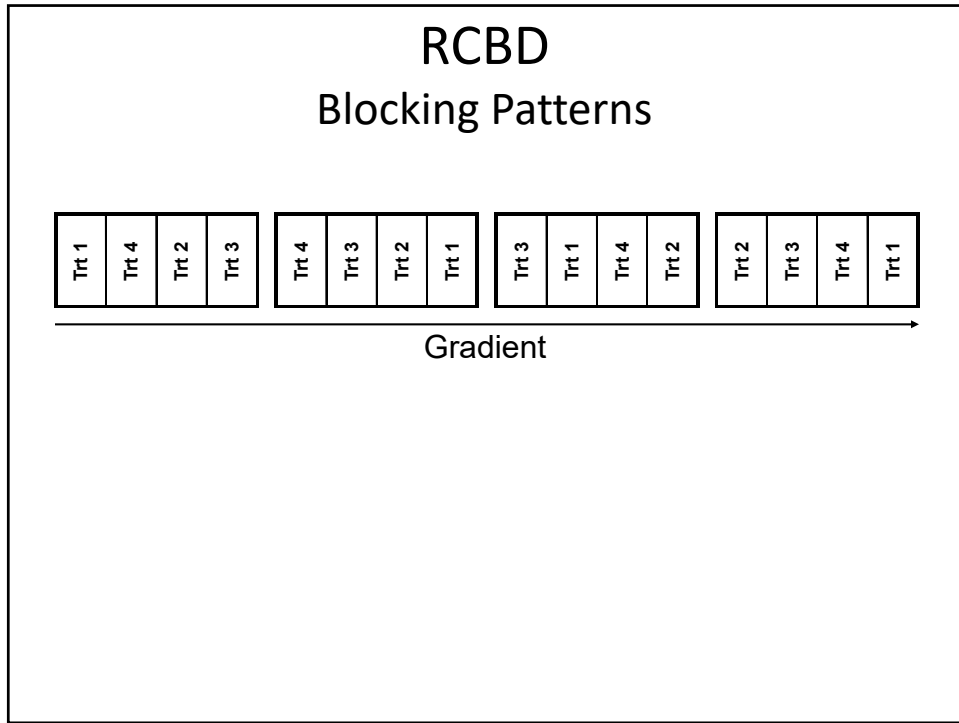
I	II	III	IV
Trt 2	Trt 1	Trt 4	Trt 1
Trt 1	Trt 3	Trt 3	Trt 4
Trt 3	Trt 4	Trt 1	Trt 2
Trt 4	Trt 2	Trt 2	Trt 3

$$Y_{ijk} = \mu + B_i + T_j + \varepsilon_{ij}$$

## RCBD ANOVA Comparison with CRD

CRD			RCBD		
Source	df	SS	Source	df	SS
$T_i$	$I - 1$	$J \sum_{i=1}^I (\bar{Y}_{i.} - \bar{Y}_{..})^2$	$B_i$	$I - 1$	$J \sum_{i=1}^I (\bar{Y}_{i.} - \bar{Y}_{..})^2$
$\varepsilon_{(ij)}$	$I(J - 1)$	$\sum_{i=1}^I \sum_{j=1}^J (Y_{ij} - \bar{Y}_{i.})^2$	$T_j$	$J - 1$	$I \sum_{j=1}^J (\bar{Y}_{.j} - \bar{Y}_{..})^2$
Total	$IJ - 1$	$\sum_{i=1}^I \sum_{j=1}^J (Y_{ij} - \bar{Y}_{..})^2$	$\varepsilon_{ij}$	$(I - 1)(J - 1)$	$\sum_{i=1}^I \sum_{j=1}^J (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2$
			Total	$IJ - 1$	$\sum_{i=1}^I \sum_{j=1}^J (Y_{ij} - \bar{Y}_{..})^2$





## RCBD

### Linear Additive Model

$$Y_{ij} = \mu + B_i + \delta_{(i)} + T_j + BT_{ij}$$

Where:

$Y_{ij}$  = Observation from the  $ij^{\text{th}}$  plot

$\mu$  = Overall mean

$B_i$  = Effect of the  $i^{\text{th}}$  block

$\delta_{(i)}$  = Restriction error associated with B

$T_j$  = Effect of the  $j^{\text{th}}$  treatment

$BT_{ij}$  = Interaction effect of the  $i^{\text{th}}$  of block with the  $j^{\text{th}}$  treatment

## RCBD

### Expected Mean Squares

Blocks Random

Source	b	t	EMS
	R	F	
	i	j	
$B_i$	1	t	$\sigma^2 + t\sigma_\delta^2 + t\sigma_B^2$
$\delta_{(i)}$	1	t	$\sigma^2 + t\sigma_\delta^2$
$T_j$	b	0	$\sigma^2 + \sigma_{BT}^2 + b\Phi(T)$
$BT_{ij}$	1	0	$\sigma^2 + \sigma_{BT}^2$

## RCBD Example Kenaf Variety Trial

Variety	Block				Mean
	1	2	3	4	
Dowling	4.3	3.7	4.0	2.8	<b>3.7</b>
Evergl41	3.4	3.7	3.9	3.3	<b>3.6</b>
Evergl71	3.6	3.0	3.7	3.6	<b>3.5</b>
Gregg	3.7	4.2	4.3	2.8	<b>3.7</b>
PF2	2.6	2.5	3.2	1.7	<b>2.5</b>
SF459	3.2	5.3	2.2	2.2	<b>3.2</b>
T2	5.4	6.1	4.2	4.6	<b>5.1</b>
Whitten	4.5	3.4	3.3	2.6	<b>3.4</b>
<b>Mean</b>	<b>3.8</b>	<b>4.0</b>	<b>3.6</b>	<b>2.9</b>	<b>3.6</b>

## RCBD Example Kenaf Variety Trial ANOVA

### CRD

Source	DF	SS	Mean Square	F Value	Pr > F
Variety	7	14.53005	2.075721	3.40	0.0115
Error	24	14.66376	0.61099		

### RCBD

Source	DF	SS	Mean Square	F Value	Pr > F
Block	3	5.161725	1.720575		
Var	7	14.53005	2.075721	4.59	0.003
Error	21	9.502032	0.452478		

## RCBD Example Kenaf Variety Trial

### CRD

$$LSD_{.05,24df} = 2.063 \sqrt{\frac{2(.611)}{4}} = 1.14$$

### RCBD

$$LSD_{.05,21df} = 2.079 \sqrt{\frac{2(.452)}{4}} = 0.99$$

## RCBD

### Relative Efficiency

$$RE = \frac{(n_B + 1)(n_C + 3)MS_{eC}}{(n_C + 1)(n_B + 3)MS_{eB}}$$

Where:  $MS_C$  = Error mean square for CRD

$MS_B$  = Error mean square for RCBD

$n_C$  = error df for CRD

$n_B$  = error df for RCBD

### RCBD Example Kenaf Variety Trial

$$RE = \frac{(21+1)(24+3)0.611}{(24+1)(21+3)0.453} = 1.34$$

Where:  $MS_C = 0.611$

$MS_B = 0.453$

$n_C = 24$

$n_B = 21$

### RCBD ANOVA

#### Analysis – Barley Example Petersen, 1994

Treatment	Blocks (Soil Types)				Mean
	1	2	3	4	
1 $(\text{NH}_4)_2\text{SO}_4$	32.1	35.6	41.9	35.4	36.25
2 $\text{NH}_4\text{NO}_3$	30.1	31.5	37.1	30.8	32.38
3 $\text{CO}(\text{NH}_2)_2$	25.4	27.1	33.8	31.1	29.35
4 $\text{Ca}(\text{NO}_3)_2$	24.1	33.0	35.6	31.4	31.03
5 $\text{NaNO}_3$	26.1	31.0	33.8	31.9	30.70
6 Control	23.2	24.8	26.7	26.7	25.35

## RCBD ANOVA

### Analysis – Barley Example

#### RCBD

Source	DF	SS	MS	F	P > F
Block	3	192.748	64.249	21.1	<.0001
Treatment	5	256.153	51.231	16.9	<.0001
Error	15	45.617	3.041		
Total	23	494.518			

#### CRD

Source	DF	SS	MS	F	P > F
Treatment	5	256.153	51.231	3.9	0.0148
Error	18	238.365	13.243		
Total	23	494.518			

## RCBD ANOVA

### Analysis – Barley Example

#### RCBD

$$LSD_{.05, 15 df} = 2.132 \sqrt{\frac{2(3.041)}{4}} = 2.63$$

#### CRD

$$LSD_{.05, 18 df} = 2.101 \sqrt{\frac{2(13.243)}{4}} = 5.41$$



## RCBD ANOVA

### Analysis – Barley Example

$$Efficiency = \frac{(15 + 1)(18 + 3)13.243}{(18 + 1)(15 + 3)3.041} = 4.28$$

Where:  $MS_C = 13.243$

$MS_B = 3.041$

$n_C = 18$

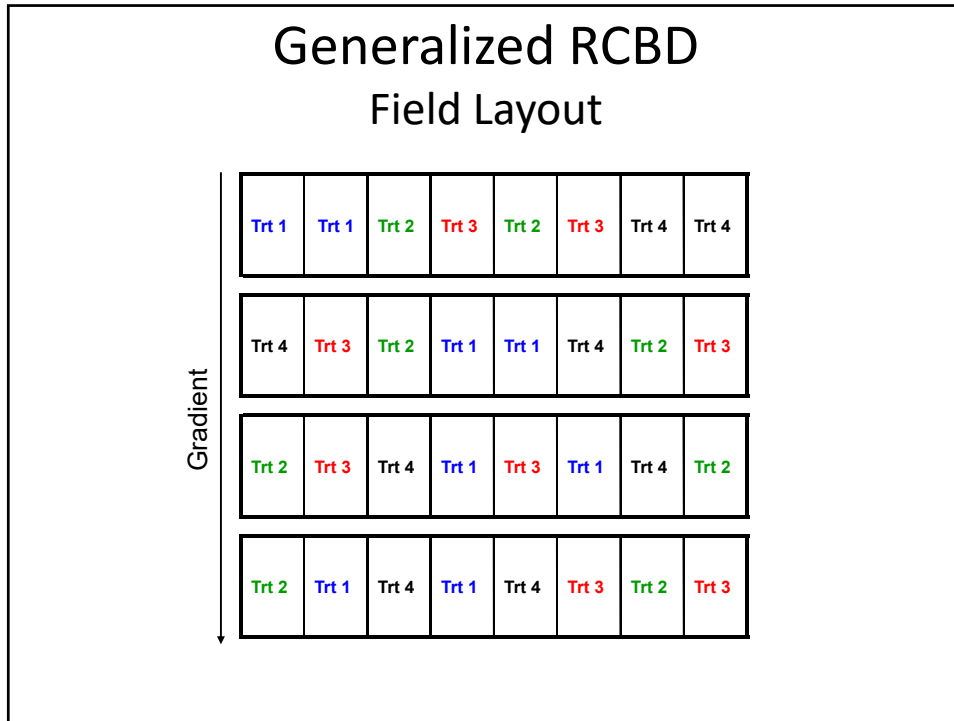
$n_B = 15$

## RCBD

### Expected Mean Squares

Blocks Fixed

Source	b F i	t F j	EMS
$B_i$	0	t	$\sigma^2 + t\sigma_\delta^2 + t\Phi(B)$
$\delta_{(i)}$	1	t	$\sigma^2 + t\sigma_\delta^2$
$T_j$	b	0	$\sigma^2 + b\Phi(T)$
$BT_{ij}$	0	0	$\sigma^2 + \Phi(BT)$



### Generalized RCBD Model

$$Y_{ijk} = \mu + B_i + \delta_{(i)} + T_j + BT_{ij} + \varepsilon_{(ij)k}$$

Where:

- $Y_{ijk}$  = Observation from the  $ijk^{\text{th}}$  plot
- $\mu$  = Overall mean
- $B_i$  = Effect of the  $i^{\text{th}}$  block.
- $\delta_{(i)}$  = Restriction error associated with B
- $T_j$  = Effect of the  $j^{\text{th}}$  treatment
- $BT_{ij}$  = Interaction effect of the  $i^{\text{th}}$  of block with the  $j^{\text{th}}$  treatment.
- $\varepsilon_{(ij)k}$  = Effect of the  $k^{\text{th}}$  eu within  $i^{\text{th}}$  block and  $j^{\text{th}}$  treatment.

## Generalized RCBD Expected Mean Squares

Blocks Random

Source	b R i	t F j	r R k	EMS
$B_i$	1	t	r	$\sigma^2 + \text{tr}\sigma_\delta^2 + \text{tr}\sigma_B^2$
$\delta_{(i)}$	1	t	r	$\sigma^2 + \text{tr}\sigma_\delta^2$
$T_j$	b	0	r	$\sigma^2 + r\sigma_{BT}^2 + br\Phi(T)$
$BT_{ij}$	1	0	r	$\sigma^2 + r\sigma_{BT}^2$
$\varepsilon_{(ij)k}$	1	1	1	$\sigma^2$

## Generalized RCBD Expected Mean Squares

Blocks Fixed

Source	b F i	t F j	r R k	EMS
$B_i$	0	t	r	$\sigma^2 + \text{tr}\sigma_\delta^2 + \text{tr}\Phi(B)$
$\delta_{(i)}$	1	t	r	$\sigma^2 + \text{tr}\sigma_\delta^2$
$T_j$	b	0	r	$\sigma^2 + br\Phi(T)$
$BT_{ij}$	0	0	r	$\sigma^2 + r\Phi(BT)$
$\varepsilon_{(ij)k}$	1	1	1	$\sigma^2$

**Fields as Blocks**  
pp. 95-103, Lorenzen and Anderson

Layout:

		Field									
		1					2				
		1	2	3	4	5	1	2	3	4	5
Fertilizer		1	2	3	4	5	1	2	3	4	5
Plot		1	6	.	.	.	26	31	.	.	.
		2	7	.	.	.	27	.	.	.	.
		3	8	.	.	.	.	.	.	.	.
		4	9	.	.	24	.	.	.	.	49
		5	10	.	20	25	.	.	.	45	50

**Fields as Blocks**  
pp. 95-103, Lorenzen and Anderson

$$Y_{ijk} = \mu + F_i + \delta_{(i)} + N_j + FN_{ij} + \varepsilon_{(ij)k}$$

Source	2 R i	5 F j	5 R k	EMS
$F_i$	1	5	5	$\sigma^2 + 25\sigma_\delta^2 + 25\sigma_F^2$
$\delta_{(i)}$	1	5	5	$\sigma^2 + 25\sigma_\delta^2$
$N_j$	2	0	5	$\sigma^2 + 5\sigma_{FN}^2 + 10\Phi(F)$
$FN_{ij}$	1	0	5	$\sigma^2 + 5\sigma_{FN}^2$
$\varepsilon_{(ij)k}$	1	1	1	$\sigma^2$